Teaching Mathematics with Technology (TMT)

Video – Introduction to Limits of a Function

Opening Webcam

Hello, welcome to the video series on teaching mathematics with technology.

This is Neeta Jose from IT for Change and one of the co-facilitators of this course. In this video, we will explore the ***limits of a function***. Before you begin to watch this video do watch our previous video on ***Linear equation and Simultaneous equations***.

Geogebra construction

To understand the limits of a function let's briefly understand the slope of a function. The slope of a function is defined as the process of finding change in the dependent variable of the function with respect to that of the independent variable. Here, I have plotted the function **f(x) = 2x+1**, where x is taken as the independent variable plotted on the x-axis and y is taken as the dependent variable plotted on the y-axis. As you know this represents the equation of a line.

Let us now mark two points **E**(x_1, y_1) and **C** (x_2,y_2) on the line. You can view the values of x and y of these points in the Algebra view. The slope of the line can be obtained by joining these two points. The line is inclined at an angle theta (θ) with the x-axis. Thus, slope of the line is given as the tangent of angle theta; here triangle EDC the opposite side CD is given as y_2 **minus** y_1 and the adjacent side ED is given as x_2 **minus** x_1. Hence, the slope of the line is equal to (y_2-y_1) over (x_2-x_1). Now let me move the points E and C along the line. You can see that the slope of the line remains constant.

Suppose, we plot another function $y=x^2$, how do we find the rate of change of function f(x) with respect to x?

We need to plot a tangent to the curve. The slope of this line can found using the slope tool which is available under the angle icon in Geogebra. Now, observe that slope of the line changes as we move the line along the curve. Also, observe how the y-value of **point A** changes when x is equal to 3, 2, 1 and 0. We see that as x reaches zero, y also decreases to reach zero. Thus we can understand how the function f(x) changes as x changes, by finding the average rate of change of the function with respect to x. Suppose, if we want to understand how a function changes at a particular point C on the curve, then we may not be able to rely on finding through the average rate of the function. This is where the concept of limits of a function becomes useful to understand the rate of change of a function at a target point.

Limit of a function can be defined as the value L to which the function f(x) approaches as x approaches x_0. It is mathematically represented as shown here $\lim_{x \to x_0} f(x) = L$

Let us see the limit of function **f(x)= 2x+1** at the **point C** which lies on the x-y coordinates (2,5) as x tends to 2 from values greater and lesser than 2. To denote values of x nearing 2 from values lesser than 2, I have created an **integer slider LHL** (which represents the left hand limit) and **RHL** (right hand limit) when x nears 2 from values greater than 2 marked by **points A** and **H** respectively on the x-axis. To understand the construction of figures, you can find them in the *construction protocol available under View in the menu bar.* Now, by moving the sliders RHL in Geogebra we can observe that the function f(x) tends to the limit 5 as x approaches 2 from the right hand side which can be clearly seen on the number line. Similarly, by moving the sliders LHL we can observe that the function f(x) tends to the limit 5 as x approaches 2 from the left side. Thus the right hand limit and left hand limit of the function is symbolically represented as shown here $\lim_{x \to 2+0} f(x)=5$, $\lim_{x \to 2-0} f(x)=5$. We can also tabulate the points E and C to the *spreadsheet in Geogebra which is available under View in menu bar*. This helps us to clearly understand the limit of the function. We will upload the video (gif) on how to record values to spreadsheet subsequently. Here. we can also visualize that when x is very close to the value 2 from either sides, the function f(x) denoted by points **T** and **S** on the y-axis is very close to 5. Since the left hand and right hand limit are equal we can say that the $\lim_{x \to 2} f(x)=5$ exists at x=2.

Now, lets see another example of limits. Here I have plotted the function $f(x)=\frac{x^2-1}{x-1}$. Clearly, we know that when we plug in x=1 we get $f(1)=\frac{0}{0}$ which is neither equal to 1 nor equal to infinity and it is termed as an indeterminant in mathematics. Since the function is not defined at x=1, we apply the concept of limits to understand how the function f(x) behaves when x nears one. **Point B** on the line gives an idea of behavior of the function f(x) as x approaches one. Observe the x-y coordinates of the point B. At x=1, you can clearly see that point B disappears and I have represented this as a hole, by styling another point. This shows that limit of this function f(x) does not exist at x=1.

Thus Geogebra is an important tool to help us visualize the behaviour of the function through the graphical representation. You can also find the limits of the function of your choice using Geogebra and interpret the behavior of the function at various points.

We will see the application of limits in our next video.
Thank You