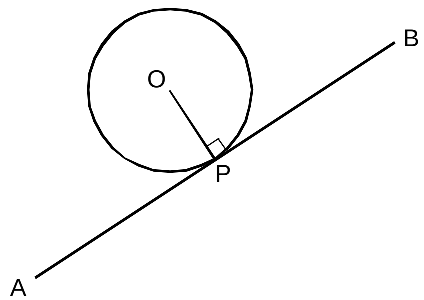
22 Tangents

After studying this lesson you will acquire knowledge about the following :

- Properties of angles related to tangents.
- Properties of tangents drawn to a circle from an external point and their applications.
- The angle between the tangent and a chord at the point of contact and the angle in the alternate segment and its application.

22.1 Tangent and the point of contact



When a straight line cuts a circle, if the two intersecting points coincide, then the straight line is a tangent to the circle. This common point is called the points of contact. According to the above diagram, the straight line AB, touches the circle with centre O, at P.

Given below are some important results connecting circles and their tangents.

(A) The shortest distance from the centre of a circle to a tangent drawn to the circle, is the radius at the point of contact.

(B) The line drawn perpendicular to a chord at a point on the circle is a tangent to the circle.

(C) The converse of the theorem (B) is as follows. The tangent drawn to a circle at any point is perpendicular to the chord drawn at that point.

(D) All the points on a tangent, other than the point of contact, lie outside the circle.

22.1 Activity

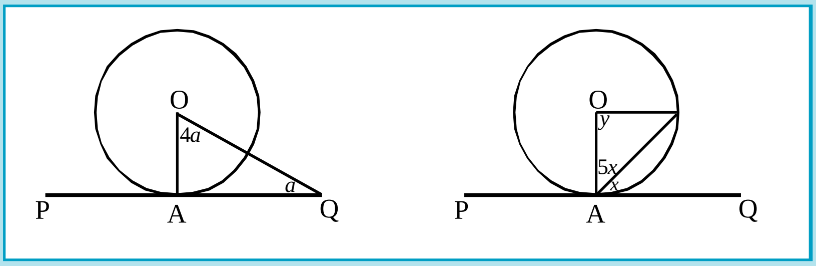
- Draw a circle with centre O and radius 3 cm.
- Mark point A on the circle
- Join OA
- Construct a perpendicular to radius OA at A.
  Name it as PA, and produce PA to Q
- Mark a point X on the line PAQ
- Join OX
- Measure and find the length of OX.
  Can it be written as $OX > OA$?
- Compare the distance from the centre to any point on the line PAQ, other than A, with the radius.

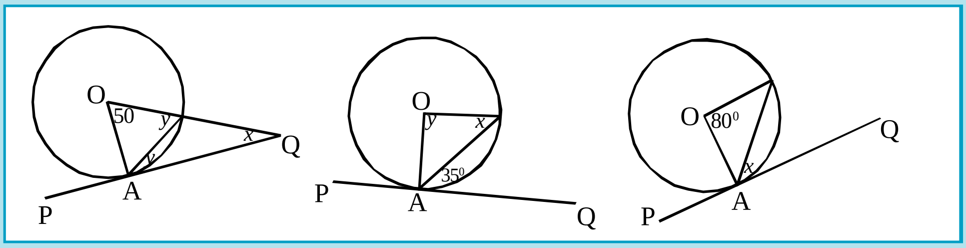
22.2 Activity

- Draw a circle with radius 5 cm on a bristol board and mark its centre as O.
- Mark any point A outside the circle and fix a pin at that point. Tie a piece of thread to the pin and fix another pin at the other end of the thread. Keep the thread taut and move it, when the thread just touches the circle, mark that point as P.
- Join OP. Measure and find the angle $O\hat{P}A$

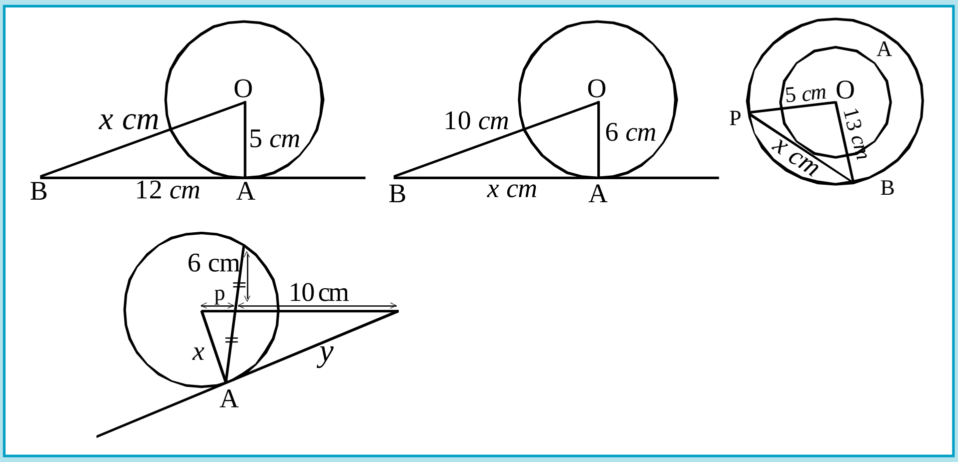
Exercise 22.1

(1) In each diagram given below, PAQ is a tangent to the circle. Using the data given in the diagrams find the values of angles marked by letters.

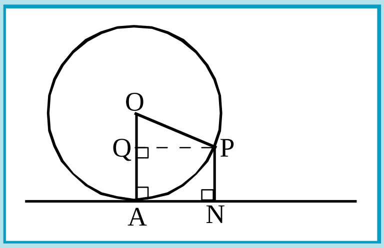




(2) In each circle below, the centre is O and a tangent is drawn at the point A. Find the distances marked by symbols.

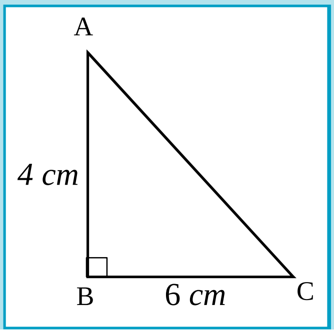


(3) AN is the tangent drawn at A to the circle with centre O. $A\hat{N}P = 90^0$
If AN = 15 cm and PN = 9 cm.
(i) Find the length of PQ
(ii) Find the length of AQ
(iii) Find the length of OQ if the radius of the circle is r
(iv) Find the radius of the circle



(4) In Δ ABC, AB = 4 cm, BC = 6 cm and $A\hat{B}C = 90^0$
To find the radius of a circle which touches AB at A,

(i) Mark point O such that $B\hat{A}O = 90^0$



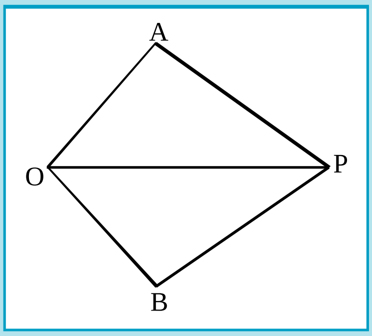
(ii) Mark K on AO produced such that $B\hat{C}K = 90^0$

(iii) Join OC

(iv) Find the length of OK taking the radius of the circle AO = r

(v) Find the value of r applying Pythagoras' theorem to Δ COK

(5) The diagram shows two right angled triangles OAP and BOP. Here OA = OB.



(i) Show that Δ OPA and Δ BOP are congruent

(ii) Name an angle equal to $O\hat{P}A$

(iii) Name an angle equal to $A\hat{O}P$

(iv) Name a side equal in length to AP

(6) O and C are centres of two given circles. OC produced cuts the circle with centre C at T. The two circles intersect at A and B. $A\hat{O}B = 60^0$

(i) Join OA, OB, TA, TB

(ii) Show that Δ OAT and Δ OBT are congruent

(iii) Name a side equal to the side TA

(iv) Name an angle equal to angle $O\hat{T}A$

(v) Name an angle equal to $T\hat{O}A$

22.2 Tangents drawn to a circle from an external point

22.3 Activity

- Draw a circle with radius 3 cm. Mark its centre as O
- Mark two points B and C on the circle, which are not on the same diameter. Draw two tangents to the circle at these points.

- Mark the point of contact of the two tangents as A
- Join OA
- Cut out the triangles AOB and AOC thus formed
- See whether the two circles coincide
- State the conclusion that can be made from this

Two tangents can be drawn to a circle from an external point. The theorem given below related to these tangents is very important.

Theorem

When two tangents are drawn to a circle from an external point, then

(i) two tangents are equal in length

(ii) the angles subtended by the tangents at the centre of the circle are equal

(iii) the line joining the centre to the external point bisects the angle between the tangents.

Data : AB and AC are tangents to the circle with centre O. B and C are the points of contact.

To prove that: (i) AB = AC

(ii) $A\hat{O}B = A\hat{O}C$

(iii) $O\hat{A}B = O\hat{A}C$

Proof : $O\hat{B}A = O\hat{C}A = 90^0$ (Tangent is perpendicular to the radius)

In right angled triangles OBA and OCA

OB = OC (radius)

OA = OA (common)

$\therefore \Delta\ OAB \equiv \Delta\ OCA$ (R.H.S)

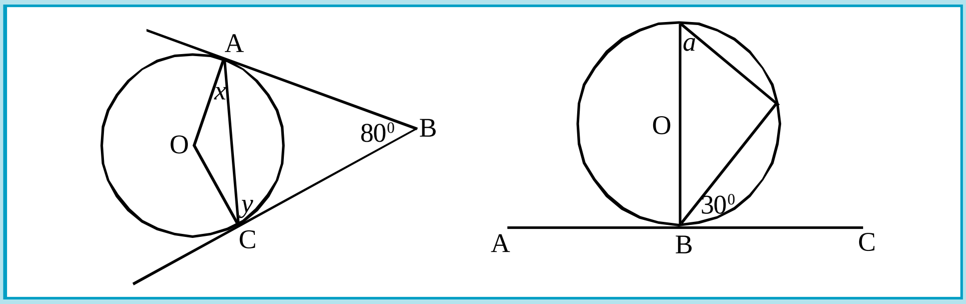
$\therefore$ AB = AC

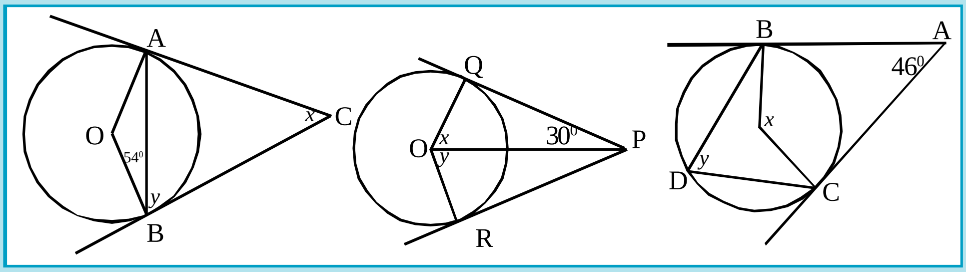
$A\hat{O}B = A\hat{O}C$

and $O\hat{A}B = O\hat{A}C$

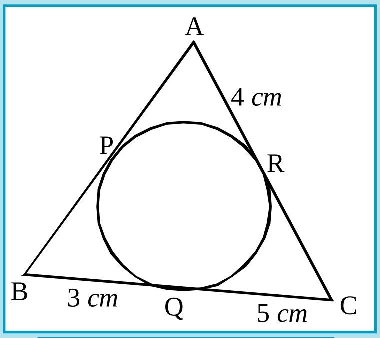
Exercise 22.2

(1) In each diagrams, the tangents drawn to a circles from an external point are shown. Find the angles marked by symbols.



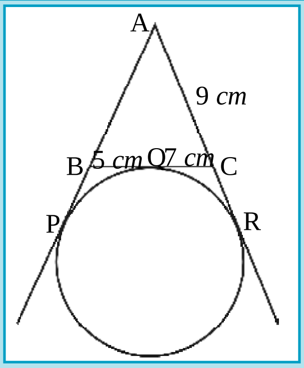


(2) AB, BC and CA are tangents to a circle.



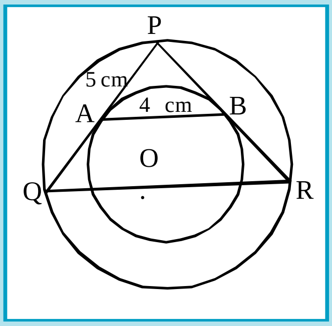
- (i) Find the length of AP
- (ii) Find the length of BP
- (iii) Find the length of CR
- (iv) Find the perimeter of the triangle ABC

(3) AP, AR and BC are tangents to a circle



- (i) Find the length of BP
- (ii) Find the length of CR
- (iii) Find the length of AR
- (iv) Find the length of AP
- (v) Find the length of AB

(4) O is the centre of two concentric circle. PQ and PR are the two tangents to the small circle. P, Q and R are point on the big circle.



(i) Find the value of $O\hat{A}P$

(ii) Find the length of AQ
Write the result used to find the length of AQ

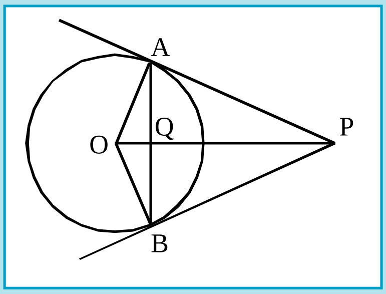
(iii) Find the length of PB

(iv) Find the length of BR

(v) Find the length of QR
Write the result used to find QR

(vi) Find the perimeter of Δ PQR

(5) PA and PB are the tangents to the circle with centre O.



(i) Show that $P\hat{A}Q = P\hat{B}Q$

(ii) Show that AQ = BQ

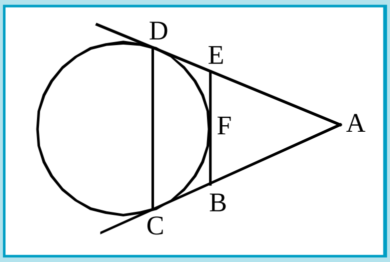
(iii) Show that $P\hat{Q}A = P\hat{Q}B$

(iv) Find the value of the angles

$P\hat{Q}A$ and $P\hat{Q}B$

(v) Is OP, the perpendicular bisector of AB ?

(6) As shown in the diagram, CBA, DEA and BFE are tangents to the circle at the points C, D, and F



(i) Write AD as the sum of two sements of a line

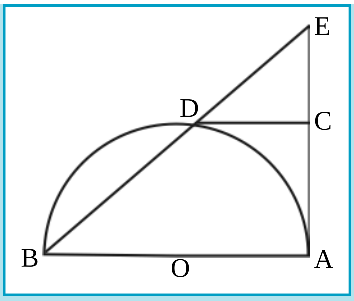
(ii) Write AC as a sum of two lengths

(iii) Name a line equal in length to ED

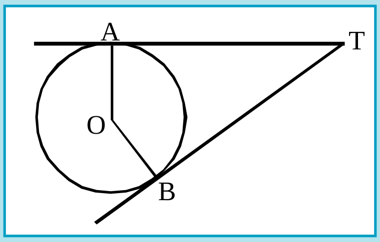
(iv) Name three segments of a line equal in length to BC

(v) Show that AD + AC = AB + BE +AE

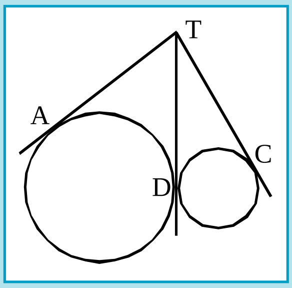
(7) BDA is a semicircle with diameter AB. The centre is O AC and CD are two tangents drawn at the points A and D respectively. Show that CA = DC = DF



(8) TA and TB are two tangents drawn to the circle with centre O from an external point T. Prove that AOBT is a cyclic quadrilateral.

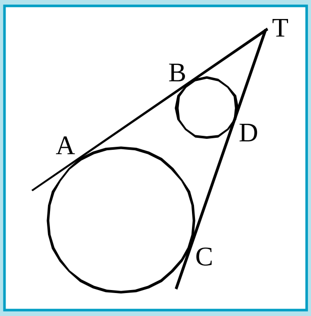


(9) As shown in the diagram, TB is the common tangent to the two circles. TA is a tangent to the big circle and TC is a tangent to the small circle.
Prove that TA = TC

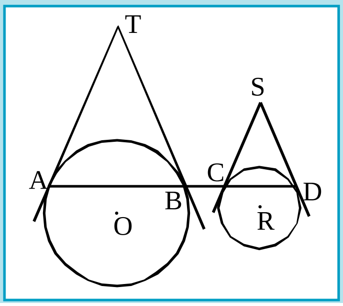


(10)EAB and EDC are two common tangents to the two circles as shown in the diagram. Prove that

(i) AB = CD
(ii) AC // BD

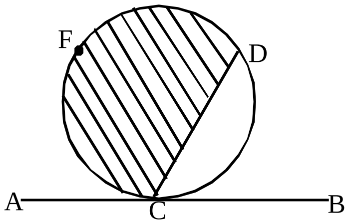


(11)As show in the diagram, TA and TB are the tangents to the circle with centre O and SC and SD are the tangents to the circle with centre R.
ABCD is a straight line.
If AT // CS, Prove that BT // DS

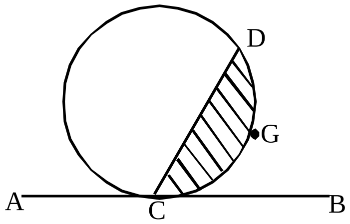


22.3 Angles in the alternate segment

(i) The shaded part of the circle, opposite angle $B\hat{C}D$ is the alternate segment corresponding to $B\hat{C}D$.



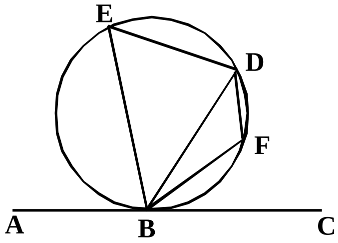
(ii) The shaded part of the circle opposite $A\hat{C}D$ is the alternate segment corresponding to angle $A\hat{C}D$.



Now we will see what is meant by an alternate segment and an angle in the alternate segment in a circle.

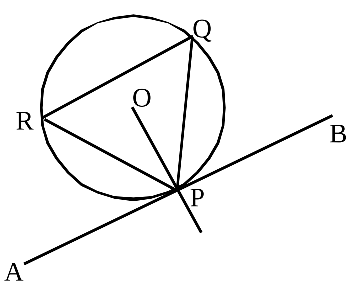
(iii) In the figure below, ABC is a tangent to the circle The point of contact is B. BD is a chord. BD divides the circle into two segments.

- $D\hat{E}B$ is the angle in the alternate segment to angle $D\hat{B}C$
- $D\hat{F}B$ is the angle in the alternate segment to angle $D\hat{B}A$



22.4 Activity

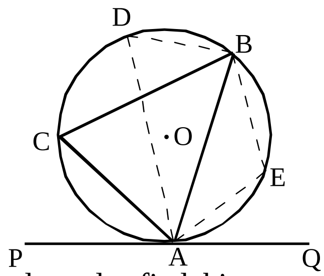
- Draw a circle with centre O and radius 5 cm on a bristol board
- Mark a point P on the circle. Draw a tangent at P. Name it as APB
- Draw a chord PQ
- In the alternate segment of $Q\hat{P}B$ mark a point R on the circumference
- Complete the triangle PQR
- Cut out the angle $Q\hat{P}B$ and keep it on Δ PQR, it will coincide
- What is the conclusion that can be made from the result ?



22.5 Activity

The centre of a circle is O. The tangent PAQ touches the circle at A.

$Q\hat{A}B = 60^0$



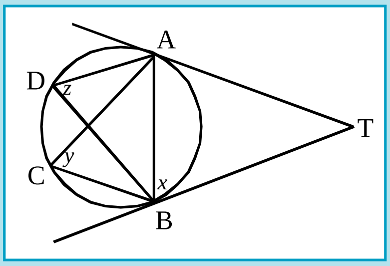
(i) What is the value of $D\hat{A}Q$.
Write the reasons
(ii) Find the value of $D\hat{A}B$
(iii) Find the value of $D\hat{B}A$
(iv) Find the value of $B\hat{D}A$
(v) What is the value of $B\hat{C}A$. Write the result used to find this
(vi) Are the angles in the alternate segment of angle $B\hat{A}Q$ equal ?
(vii) What is the value of $B\hat{E}A$. Write the result used to find this
(viii) Find the value of $B\hat{A}P$
(ix) Are the angles in the alternate segment of $B\hat{A}P$ equal ?

Theorem

The angles made by a tangent to a circle with a chord drawn from the point of contact are respectively equal to the angles in the alternate segment of the circle.

Exercise 22.3

(1) Tangents drawn to a circle at A and B intersect at T.



(i) Name a side equal to BT

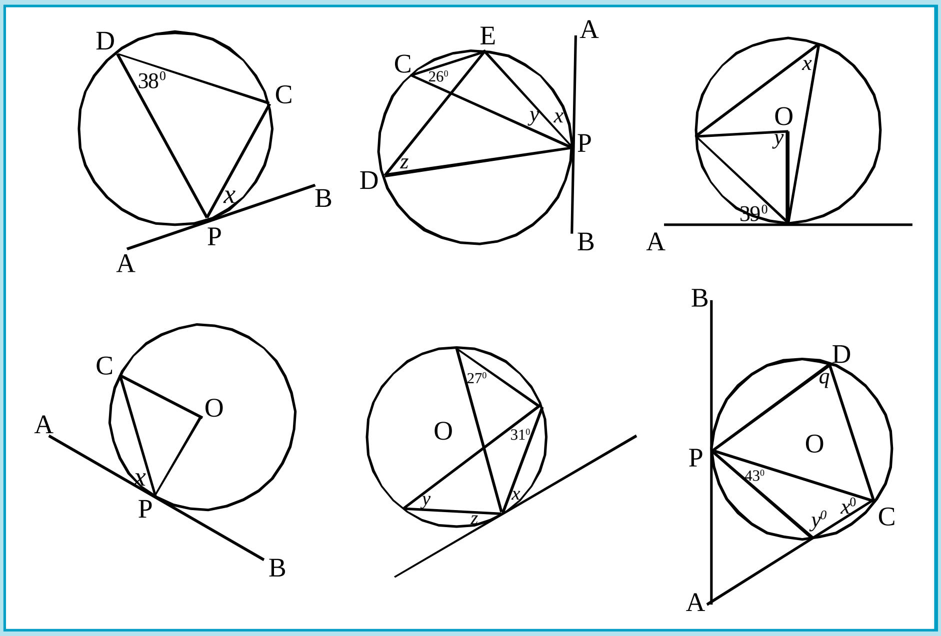
(ii) What type of a triangle is Δ ATB ?

(iii) What is the value of x ?

(iv) Name the angles in the alternate segment to $A\hat{B}T$

(v) Find the value of y and z

(2) In each diagram given below the tangent drawn at the point P on the circle with centre O is APB. Using data given in each diagram, find the value of angles denoted by symbols.

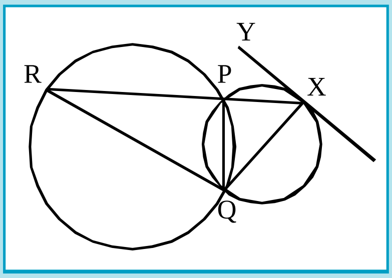


(3) QX is a tangent to the circle at Q. XY is a tangent at X.

(i) Show an angle equal to angle $Y\hat{X}P$ giving reasons

(ii) Show an angle equal to angle $P\hat{Q}X$ giving reasons

(iii) Show that QR // XY

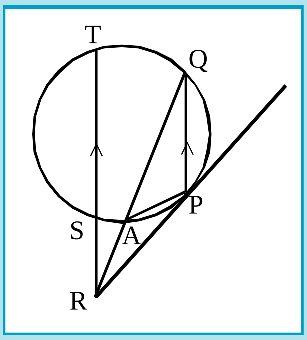


(4) PR touches the circle at P. PQ // RT

(i) Name an angle equal to angle $R\hat{P}A$. Show the result used

(ii) Name an angle equal to angle $P\hat{Q}A$

(iii) Show that $R\hat{P}A = T\hat{R}Q$



(5) ABDC is a cyclic quadrilateral. AD is the bisector of angle $\hat{A}$. DE is the tangent drawn at D.

(i) Name two angles in the alternate segment to angle $B\hat{D}E$

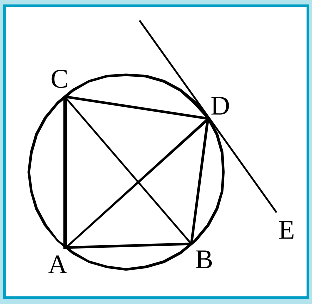
(ii) If $B\hat{D}E = x^0$, name two angles equal to x in the alternate segment

(iii) As AD bisects the angle $\hat{A}$, name an angle equal to $B\hat{A}C$

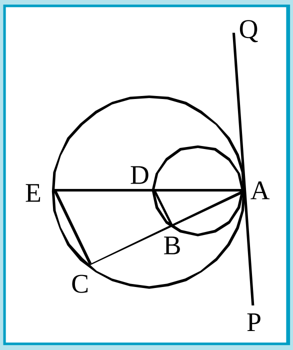
(iv) Name an angle equal to angle $D\hat{A}C$

(v) Are the angles $D\hat{B}C$ and $B\hat{C}D$ equal

(vi) Can we say that BD // CA ?



(6) Two circles touch internally at A. The common tangent at A is PAQ.



(i) What is the angle equal to $Q\hat{A}D$ in the small circle ?

(ii) What is the angle equal to $Q\hat{A}D$ in the big circle ?

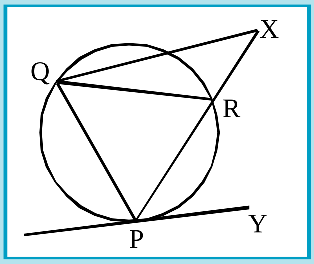
(iii) What is the angle equal to angle $A\hat{B}D$?

(iv) Can you say that BD // CE ?

(v) If AB = BC, show that the mid point of AE is D

(vi) If it is given that AB = BC and $D\hat{A}Q = 90^0$, show that the centre of the big circle is D

(7) In the diagram the tangent PY touches the circle at P. PY // QX. Show that $P\hat{Q}X = P\hat{X}Q$



(8) K, L and M are point on a circle. The tangent drawn at L meets KM produced at P.

Prove that $K\hat{L}P = L\hat{M}P$